Econ 802

First Midterm Exam

Greg Dow October 9, 2019

All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Partially Observable Enterprises uses production plans of the form $y = (y_1, y_2)$ where $y_1 \leq 0$ is an input and $y_2 \geq 0$ is an output. Price vectors have the form $p = (p_1, p_2) > 0$. You observe the firm's production plans y^A and y^B in periods A and B but you do <u>not</u> observe the corresponding price vectors $p^A > 0$ and $p^B > 0$.

(a) Are there any production plans (y^A, y^B) that would <u>never</u> be consistent with profit maximization for <u>any</u> prices (p^A, p^B)? What is needed for (y^A, y^B) to be consistent with profit maximization for <u>some</u> prices (p^A, p^B)? Use a graph to explain.

(b) Assume (y^A, y^B) are consistent with profit maximization for some prices (p^A, p^B) and that the firm does maximize profit. What restrictions do your observations of (y^A, y^B) impose on the true price vector p^A? What restrictions do they impose on the true price vector p^B? Use a graph to explain.

(c) Make the same assumptions as in (b). Define YO to be the <u>largest</u> production possibility set that is monotonic, convex, closed, and includes (y^A, y^B). In general this set depends on the prices (p^A, p^B). If you don't observe the prices but you do know that $p^A = p^B$, can you determine the set YO? Use a graph to explain.

2. A firm has the production function $f(x)$ where the input vector is $x = (x_1 \,.\,.\, x_n) \geq 0$. The output price is $p > 0$ and the input prices are $w = (w_1 \,.\,.\, w_n) > 0$. Let $x_i(p, w)$ be the unconditional demand function for input i.

(a) Suppose the Hessian matrix for $f(x)$ is negative definite at all $x \geq 0$. What can you say about how $x_i(p, w)$ responds to a change in w_i? Carefully justify your answer.

(b) Suppose you do not know anything about the Hessian matrix, but you know the profit function $\pi(p, w)$ is differentiable. What can you say about how $x_i(p, w)$ responds to a change in w_i? Carefully justify your answer.

(c) Suppose you cannot make any assumptions about differentiability. What can you say about how x_i responds to a change in w_i? Carefully justify your answer.

3. Linear Enterprises has the production function $f(x) = ax$ where $x = (x_1 \ldots x_n) \geq 0$ is the input vector and $a = (a_1 \ldots a_n) > 0$ is a vector of technological parameters. The output price is $p > 0$ and the input prices are $w = (w_1 \ldots w_n) > 0$.

(a) Under what conditions (if any) does the firm's profit maximization problem have a solution? If this problem does have a solution, under what conditions (if any) is the solution unique? Explain your reasoning carefully.

(b) Consider $n = 2$. Under what conditions (if any) does the firm's cost minimization problem have a solution? Explain your reasoning carefully using graphs.

(c) Consider the general case $n \geq 2$. Assume the cost minimization problem has a solution and find the cost function $c(w, y)$. Explain your reasoning carefully.

4. Exponential Enterprises has the production function $f(x) = 1 - e^{ax}$ where $x \geq 0$ is a scalar input and $y = f(x)$ is a scalar output.

(a) Does this technology make economic sense when $a < 0$? What about $a = 0$? What about $a > 0$? Explain briefly in each case.

(b) Assume $a < 0$. The output price is $p > 0$ and the input price is $w > 0$. Set up the profit maximization problem with a Kuhn-Tucker multiplier for the constraint $x \geq 0$. Then identify the conditions under which the firm would choose $x = 0$ or $x > 0$. When $x > 0$, what is the input demand function $x(p, w)$? Note: ignore the second order conditions and just use the first order conditions.

(c) Compute the firm's cost function $c(w, y)$. Then draw a graph showing the shape of the marginal cost curve. Explain carefully.

5. Here are some miscellaneous questions.

(a) Write a production plan as $y = (y_1 \ldots y_n)$ where outputs are positive and inputs are negative. Let the price vector be $p = (p_1 \ldots p_n) > 0$. Prove that the profit function $\pi(p)$ is linearly homogeneous.

(b) Using the same notation as in part (a), prove that $\pi(p)$ is convex.

(c) Consider a Cobb-Douglas function with two inputs. For a given output $y > 0$, the firm minimizes cost. Calculate the cost share $z = w_1x_1/(w_1x_1 + w_2x_2)$ for input 1 and show that z does not depend on the prices w or the output level y. Interpret this result.